Homework Assignment # 1

**Solution 1:-**

P(A U B)= P(A)+P(B)-P(A n B)

**Proof: -** Refer to Figure. Both A U B and A n B= (Ac U Bc)c are events because A and B are events. Similarly, A n Bc and B n Ac are also events.

Notice that A n Bc , B n Ac, and A N B are pairwise disjoints events. Hence,

P(A) + P(B) – P(A n B)

= P ((A n Bc ) U (A n B)) + P ((B n Ac ) U (A n B)) – P (A n B)

= P (A n Bc ) + P (A n B) + P (B n Ac ) + P (A n B) – P (A n B)

= P (A n Bc ) + P (A n B) + P (B n Ac )

= P((A n Bc) U (A U B) U (B n Ac))

= P(AUB)

Hence Proved P (A)+ P (B)- P(A n B) = P(AUB).

**Solution 2:-** P(A)=P (A|B)+ P(A|Bc )

This expression is true and can be proved easily by following example:-

Consider two fair dices are rolled and both dices show 6.

Let A: - First dice shows 6

B: - Second dice shows 6

P(A)= 1/6

P(B)= 1/6

P(A n B )= 1/36

P(A n B)

P(A/B)= \_\_\_\_\_\_\_

P (B)

1/36

= \_\_\_\_

1/6

= 1/6

P(A n BC)

P(A/Bc)= \_\_\_\_\_\_\_\_

P(Bc)

P (Probability that 6 came up and )

= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

P ( Not getting six on other dice)

0

= \_\_\_\_\_\_

5/6

= 0

Hence **P(A)=P (A|B)+ P(A|Bc** **)**.

**Solution 3: -**

1. Let p be the additional wins of Player 1 and q be the additional wins for Player 2

Probability (Player 1 Wining) = P (Player one gets 8 Heads before Player 2 gets 8 tails)

= P ( p=8-4 before q=8-6 ) ( Mentioned in problem )

= P ( p= 4 before q=2)

**Case (i):-**  P( p=4 and L=0) i.e 4 Heads in a row

1

= 1/24= \_\_

16

**Case (ii):-** P( p=4 and q=1)

= {THHHH, HTHHH, HHTHH, HHHTH} **Note:-**HHHHHT is not to be considered

1

= 4 x \_\_\_

25

1

= \_\_\_

8

Probability (Player one wins) = Case ( i ) + Case ( ii )

1 1

= \_\_\_\_ + \_\_\_

16 8

3

= \_\_\_

16

1. General Expression for n, m and l

Assuming 0≤ m< n and 0 ≤ l <n.

Let p be number of wins by Player One

Let q be number of wins by Player Two

P ( Player one wins) = P (p=n before q=n)

= P [ (p=n and q=l) OR (p=n and q= l+1) OR (p=n and q=n-1) ]

= ∑n-1i=1  P (p=n and q=i)

Now 0 ≤ i< n

First, We need to complete a win by player one which implies getting n-m more heads.

Since tosses are independent, so this probability is (1/2)n-m .

Next, There will be i tails which gives a probability of (1/2) i.

Finally, tails can occur in i of exactly n-m+1 different places to have heads and complete a win, to which we must add i places for the fact that last toss must always be head. Since order does not matter, there are (n-m+i-1 i ) places to put i tails. Putting all this together, we conclude that for given i:

P ( p=n and q=i) = n-m+i-1Ci (1/2)n-m (1/2) i

= n-m+i-1Ci (1/2)n-m (1/2)n-m+i

Final answer is sum of all of i where 0 ≤ i< n- (l+1).

Hence P (Player One Wins) = ∑n-(l+1) i=0  P (p= n and q=i)

= ∑n-(l+1) i=0  n-m+i-1Ci  (1/2)n-m+i

Veracity of this expression can be tested by substituting values from (a) and checking result

n=8 , m=4 and l=6

= ∑i=0  3Ci (1/2)4+i = ∑i=0  3C0 (1/2)4+0

= 3C0 (1/2)4+0 + 4C1 (1/2)4+1

= 3C0 (1/2)4 + 4C1 (1/2)5

= 1 x (1/16)+ 4 x (1/32) = 3/16

Hence General expression derived holds good.

**Solution 6:-**

P x|y (X=x | Y= y) = ∑ z€Ωz Px|yz (X=x |Y=y, Z= z) Pz|y (Z=z | Y=y)

This expression holds good.

Let us start with Right hand side of the equation,

∑ z€Ωz Px|yz (X=x |Y=y, Z= z) Pz|y (Z=z | Y=y)

∑ z€Ωz Px|yz (X=x ,Y=y, Z= z)

= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ . Pz|y (Z=z | Y=y)

Py|z (Y=y | Z=z)

∑ z€Ωz Px|yz (X=x ,Y=y, Z= z)

= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ . Pz|y (Z=z | Y=y)

Pz|y (Z=z | Y=y) . Py (Y=y)

∑ z€Ωz Px|yz (X=x ,Y=y, Z= z)

= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Py (Y=y)

∑ z€Ωz Px|yz (X=x, Z=z | Y= y)

= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ . Py (Y=y)

Py (Y=y)

= ∑ z€Ωz Px|yz (X=x, Z=z | Y= y)

= Px|y (X=x,| Y= y) By total probability

Bonus Question

**Solution:-**

1. Volume of a hypersphere in n-dimensions of radius a is given by:

Πn/2 an

Ba (n) = \_\_\_\_\_\_\_\_\_\_\_ from Reference

Г (n/2+ 1)

Where Г: - Gamma Function

Volume of a hypercube in n-dimension with side 2a is given by

Ca (n)= (2a)n

Volume of Hypersphere of radius a

R(n) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Volume of Hypercube with side 2a

Πn/2 an

\_\_\_\_\_\_\_\_\_\_\_

Г (n/2+ 1)

R(n) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(2a)n

Πn/2 an

= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Г (n/2+ 1) (2a)n

Πn/2

= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Г (n/2+ 1) 2n

For High Dimensional space takes limit n to ∞